Question 3. Consider the problem of minimizing the maximum lateness of jobs scheduled on a single machine. Job j is available from r_j onwards, and has due date d_j. Consider the following instance of $1|r_j|L_{\max}$:

job j	1	2	3	4	5	6	7	8	9	10	11	12
p_j	2	1	1	3	2	3	1	2	2	4	2	1
r_j	0	1	12	7	3	9	0	7	12	9	6	17
d_j	6	2	11	20	20	16	10	8	10	12	11	22

(a) Solve the above problem, *allowing for preemption.* Give the answer by providing a preemptive schedule, and completing the following table (please mark preempted jobs):

job j	1	2	3	4	5	6	7	8	9	10	11	12
d_j	6	2	11	20	20	16	10	8	10	12	11	22
C_j												
L_j												

(b) Derive from your schedule the set $S \subseteq J$, for which the appropriate lower bound on optimal value $L_{\max}^{\text{OPT}}$ equals the realized maximum lateness. Describe the lower bound in its general form.

(c) In an *on-line* context, the jobs would have been revealed to us in order and at the time of their due dates. In order to make a reasonable non-preemptive schedule, without knowing the future, we might have chosen to form a so-called α-schedule: jobs are scheduled tentatively in a preemptive way; once their α-point has passed, they are really available for processing.

Give the definition of the alpha-point T_j^α in general, and compute for the given preemptive schedule for each job its α-point, for $\alpha = 0.5$.

Describe the result by completing the following table

job j	1	2	3	4	5	6	7	8	9	10	11	12
T_j^α												

(d) Give the resulting non-preemptive schedule based on postponing the release of a job to its alpha-point, for $\alpha = 0.5$. Describe the answer by completing the following table:

job j	1	2	3	4	5	6	7	8	9	10	11	12
d_j	6	2	11	20	20	16	10	8	10	12	11	22
C_j												
L_j												